

**HUMAN-COMPUTER INTERACTION** THIRD EDITION DIX FINLAY ABOWD BEALE

chapter 17

models of the system

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**Models of the System**

**Standard Formalisms**  
software engineering notations used to specify the required behaviour of specific interactive systems

**Interaction Models**  
special purpose mathematical models of interactive systems used to describe usability properties at a generic level

**Continuous Behaviour**  
activity between the events, objects with continuous motion, models of time

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**types of system model**

- dialogue – main modes
- full state definition
- abstract interaction model

} specific system

— generic issues

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## Relationship with dialogue

- Dialogue modelling is linked to semantics
- System semantics affects the dialogue structure
- But the bias is different
- Rather than dictate what actions are legal, these formalisms tell what each action does to the system.

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## Irony

- Computers are inherently mathematical machines
- Humans are not
- Formal techniques are well accepted for cognitive models of the user and the dialogue (what the user *should do*)
- Formal techniques are not yet well accepted for dictating what the system should do *for the user!*

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## standard formalisms

general computing notations  
to specify a particular system

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## standard formalisms

Standard software engineering formalisms can be used to specify an interactive system.

Referred to as *formal methods*

- Model based – describe system states and operations
  - Z, VDM
- Algebraic – describe effects of sequences of actions
  - OBJ, Larch, ACT-ONE
- Extended logics – describe when things happen and who is responsible
  - temporal and deontic logics

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## Uses of SE formal notations

- For communication
  - common language
  - remove ambiguity (possibly)
  - succinct and precise
- For analysis
  - internal consistency
  - external consistency
    - with eventual program
    - with respect to requirements (safety, security, HCI)
  - specific versus generic

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## model-based methods

- use general mathematics:
  - numbers, sets, functions
- use them to define
  - state
  - operations on state

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## model-based methods

- describe state using variables
- types of variables:
  - basic type:
    - $x: \text{Nat}$       – non-negative integer  $\{0,1,2,\dots\}$   
or in the Z font:  $\mathbb{N}$
  - individual item from set:
    - shape\_type: {line, ellipse, rectangle}
  - subset of bigger set:
    - selection:  $\text{set Nat}$       – set of integers  
or in the Z font:  $\mathbb{P}\mathbb{N}$
  - function (often finite):
    - objects:  $\text{Nat} \rightarrow \text{Shape\_Type}$

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## Mathematics and programs

Mathematical counterparts to common programming constructs

Programming	Mathematics
types	sets
basic types	basic sets
constructed types	constructed sets
records	unordered tuples
lists	sequences
functions	functions
procedures	relations

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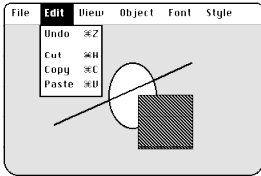
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## running example ...



a simple graphics drawing package supports several types of shape

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## define your own types

an x,y location is defined by two numbers

```
Point == Nat × Nat
```

a graphic object is defined by its shape, size, and centre

```
Shape ==
```

shape: {line, ellipse, rectangle}
x, y: Point      – position of centre
wid: Nat
ht: Nat          – size of shape

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## ... yet another type definition

A collection of graphic objects can be identified by a 'lookup dictionary'

```
[Id]
Shape_Dict == Id → Shape
```

- Id is an introduced set
  - some sort of unique identifier for each object
- Shape\_Dict is a function
  - for any Id within its domain (the valid shapes) it gives you a corresponding shape this means for any

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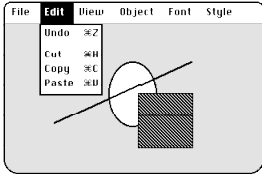
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## use them to define state



shapes:	Shape_Dict
selection:	set Id      – selected objects

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## invariants and initial state

invariants – conditions that always be true  
– must be preserved by every operation

$\text{selection} \subseteq \text{dom shapes}$   
– selection must consist of valid objects

initial state – how the system starts!

$\text{dom shapes} = \{\}$  – no objects  
 $\text{selection} = \{\}$  – selection is empty

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## Defining operations

State change is represented as two copies of the state  
before – State  
after – State'

The Unselect operation deselects any selected objects

unselect:

$\text{selection}' = \{\}$  – new selection is empty  
 $\text{shapes}' = \text{shapes}$  – but nothing else changes

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## ... another operation

delete:

$\text{dom shapes}' = \text{dom shapes} - \text{selection}$   
– remove selected objects  
 $\forall \text{id} \in \text{dom shapes}'$   
 $\text{shapes}'(\text{id}) = \text{shapes}(\text{id})$   
– remaining objects unchanged  
 $\text{selection}' = \{\}$  – new selection is empty

✍ note again use of primed variables for 'new' state

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## display/presentation

- details usually very complex (pixels etc.)  
... but can define **what** is visible

Visible\_Shape\_Type = Shape\_Type  
highlight: Bool

display:

```
vis_objects: set Visible_Shape_Type
vis_objects =
  { ( objects(id), sel(id) ) | id ∈ dom objects }
  where sel(id) = id ∈ selection
```

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## Interface issues

- Framing problem
  - everything else stays the same
  - can be complicated with state invariants
- Internal consistency
  - do operations define any legal transition?
- External consistency
  - must be formulated as theorems to prove
  - clear for refinement, not so for requirements
- Separation
  - distinction between system functionality and presentation is not explicit

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## Algebraic notations

- Model based notations
  - emphasise constructing an explicit representations of the system state.
- Algebraic notations
  - provide only implicit information about the system state.
- Model based operations
  - defined in terms of their effect on system components.
- Algebraic operations
  - defined in terms of their relationship with the other operations.

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## Return to graphics example

### types

State, Pt

### operations

init :  $\rightarrow$  State

make ellipse : Pt  $\times$  State  $\rightarrow$  State

move : Pt  $\times$  State  $\rightarrow$  State

unselect : State  $\rightarrow$  State

delete : State  $\rightarrow$  State

### axioms

for all st  $\in$  State, p  $\in$  Pt •

1. delete(make ellipse(st)) = unselect(st)

2. unselect(unselect(st)) = unselect(st)

3. move(p; unselect(st)) = unselect(st)

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## Issues for algebraic notations

- Ease of use
  - a different way of thinking than traditional programming
- Internal consistency
  - are there any axioms which contradict others?
- External consistency
  - with respect to executable system less clear
- External consistency
  - with respect to requirements is made explicit and automation possible
- Completeness
  - is every operation completely defined?

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## Extended logics

- Model based and algebraic notations make extended use of propositional and predicate logic.
- Propositions
  - expressions made up of atomic terms: p, q, r, ...
  - composed with logical operations:  $\wedge \vee \neg \Rightarrow \dots$
- Predicates
  - propositions with variables, e.g., p(x)
  - and quantified expressions:  $\forall \exists$
- Not convenient for expressing time, responsibility and freedom, notions sometimes needed for HCI requirements.

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## Temporal logics

Time considered as succession of events

Basic operators:

$\square$	- always	$\square$	(G funnier than A)
$\diamond$	- eventually	$\diamond$	(G understands A)
$\square \neg$	- never	$\square \neg$	(rains in So. Cal.)

Other bounded operators:

p until q	- weaker than $\square$
p before q	- stronger than $\diamond$

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## Explicit time

- These temporal logics do not explicitly mention time, so some requirements cannot be expressed
- Active research area, but not so much with HCI
- Gradual degradation more important than time-criticality
- Myth of the infinitely fast machine ...

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## Deontic logics

For expressing responsibility, obligation between agents  
(e.g., the human, the organisation, the computer)

permission *per*  
obligation *obl*

For example:

```
owns( Jane, file `fred' ) =>
per( Jane, request( 'print fred' ) )
performs( Jane, request( 'print fred' ) ) =>
obl( lp3, print(file `fred'))
```

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## Issues for extended logics

- Safety properties
  - stipulating that bad things do not happen
- Liveness properties
  - stipulating that good things do happen
- Executability versus expressiveness
  - easy to specify impossible situations
  - difficult to express executable requirements
  - settle for eventual executable
- Group issues and deontics
  - obligations for single-user systems have personal impact
  - for groupware ... consider implications for other users.

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## interaction models

PIE model  
defining properties  
undo

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## Interaction models

General computational models were not designed with the user in mind

We need models that sit between the software engineering formalism and our understanding of HCI

- formal
  - the PIE model for expressing general interactive properties to support usability
- informal
  - interactive architectures (MVC, PAC, ALV) to motivate separation and modularisation of functionality and presentation (chap 8)
- semi-formal
  - status-event analysis for viewing a slice of an interactive system that spans several layers (chap 18)

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## the PIE model

'minimal' black-box model of interactive system  
 focused on external observable aspects of interaction

```

  graph LR
    P[Process] -- I [Interaction] --> E[Effect]
    E -- result --> R[Result]
    E -- disp --> D[Display]
  
```

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## PIE model - user input

- sequence of commands
- commands include:
  - keyboard, mouse movement, mouse click
- call the set of commands C
- call the sequence P
  - P = seq C

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## PIE model - system response

- the 'effect'
- effect composed of:
  - ephemeral *display*
  - the final *result*
    - (e.g. printout, changed file)
- call the set of effects E

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## PIE model - the connection

- given any history of commands (P)
- there is some current effect
- call the mapping the interpretation (I)  
 $I: P \rightarrow E$

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## More formally

$[C; E; D; R]$   
 $P == \text{seq } C$

$I : P \rightarrow E$
display : $E \rightarrow D$
result : $E \rightarrow R$

Alternatively, we can derive a state transition function from the PIE.

doit : $E \times P \rightarrow E$
doit( I(p), q) = I(p q)
doit( doit(e, p), q) = doit(e, p q)

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## Expressing properties

WYSIWYG (what you see is what you get)

- What does this really mean, and how can we test product X to see if it satisfies a claim that it is WYSIWYG?

Limited scope general properties which support WYSIWYG

- Observability
  - what you can tell about the current state of the system from the display
- Predictability
  - what you can tell about the future behaviour

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## Observability & predictability

Two possible interpretations of WYSIWYG:

What you see is what you:  
*will get* at the printer  
*have got* in the system

Predictability is a special case of observability

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## what you get at the printer

```

    graph LR
      P -- I --> E
      E -- result --> R
      E -- display --> D
      D -- predict --> R
  
```

$\exists \text{ predict} \in (D \rightarrow R)$  s.t.  $\text{predict} \circ \text{display} = \text{result}$

- but really not quite the full meaning

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## stronger - what is in the state

```

    graph LR
      P -- I --> E
      E -- result --> R
      E -- display --> D
      D -- predict_E --> E
  
```

$\exists \text{ predict}_E \in (D \rightarrow R)$  s.t.  $\text{predict}_E \circ \text{display} = \text{id}_E$

- but too strong – only allows trivial systems where everything is always visible

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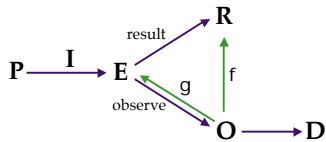
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### Relaxing the property



- O – the things you can indirectly observe in the system through scrolling etc.
- predict the result  
 $\exists f \in (O \rightarrow R)$  s.t.  $f \circ \text{observe} = \text{result}$
- or the effect  
 $\exists g \in (O \rightarrow R)$  s.t.  $g \circ \text{observe} = \text{id}_E$

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### Reachability and undo

- Reachability – getting from one state to another.  
 $\forall e, e' \in E \cdot \exists p \in P \cdot \text{doit}(e, p) = e'$
- Too weak
- Undo – reachability applied between current state and last state.  
 $\forall c \in C \cdot \text{doit}(e, c \text{ undo}) = e$
- Impossible except for very simple system with at most two states!
- Better models of *undo* treat it as a special command to avoid this problem

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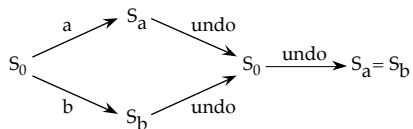
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### proving things - undo

$\forall c : c \text{ undo} \sim \text{null} ?$

only for  $c \neq \text{undo}$




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lesson

- undo is no ordinary command!
- other meta-commands:
  - back/forward in browsers
  - history window

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Issues for PIE properties

- Insufficient
  - define necessary but not sufficient properties for usability.
- Generic
  - can be applied to any system
- Proof obligations
  - for system defined in SE formalism
- Scale
  - how to prove many properties of a large system
- Scope
  - limiting applicability of certain properties
- Insight
  - gained from abstraction is reusable

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continuous behaviour

mouse movement  
status–event & hybrid models  
granularity and gestalt

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## dealing with the mouse

- Mouse always has a location
  - not just a sequence of events
  - a *status* value
- update depends on current mouse location
  - doit:  $E \times C \times M \rightarrow E$
  - captures *trajectory independent* behaviour
- also display depends on mouse location
  - display:  $E \times M \rightarrow D$
  - e.g. dragging window

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## formal aspects of status-event

- events
  - at specific moments of time
    - keystrokes, beeps, stroke of midnight in Cinderella
- status
  - values of a period of time
    - current computer display, location of mouse, internal state of computer, the weather

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## interstitial behaviour

- discrete models
  - what happens at events
- status–event analysis
  - also what happens *between* events
- centrality ...
  - in GUI – the *feel*
    - dragging, scrolling, etc.
  - in rich media – the main purpose

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## formalised ...

action:  
 user-event x input-status x state  
 -> response-event x (new) state

interstitial behaviour:  
 user-event x input-status x state  
 -> response-event x (new) state

note:  
 current input-status => trajectory independent  
 history of input-status allows freehand drawing etc.

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## status-change events

- events can change status
- *some* changes of status are meaningful events
  - ▮ when bank balance < \$100  
need to do more work!
- not all changes!
  - every second is a change in time
  - but only some times critical
    - ▮ when time = 12:30 - eat lunch
- implementation issues
  - system design - sensors, polling behaviour

more on status-event analysis in chapter 18

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## making everything continuous

- physics & engineering
  - everything is continuous
    - time, location, velocity, acceleration, force, mass

$$\frac{dx}{dt} = v \quad \frac{dv}{dt} = -g \quad x = vt - \frac{1}{2}gt^2$$

- can model everything as pure continuous
  - state<sub>t</sub> =  $\varphi ( t, t_0, \text{state}_{t_0}, \text{inputs during } [t_0, t] )$
  - output<sub>t</sub> =  $\eta ( \text{state}_t )$
  - like interstitial behaviour
- but clumsy for events - in practice need both

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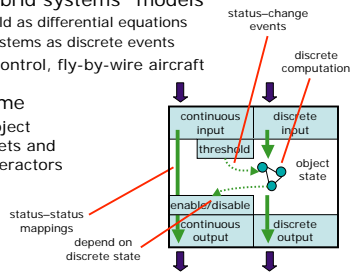
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## hybrid models

- computing "hybrid systems" models
  - physical world as differential equations
  - computer systems as discrete events
  - for industrial control, fly-by-wire aircraft
- adopted by some
  - e.g. TACIT project
  - Hybrid Petri Nets and continuous interactors




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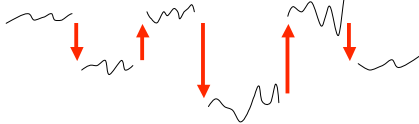
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## common features

- actions
  - at events, discrete changes in state
- interstitial behaviour
  - between events, continuous change




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## granularity and Gestalt

- granularity issues
  - do it today
    - » next 24 hours, before 5pm, before midnight?
- two timing
  - 'infinitely' fast times
    - » computer calculation c.f. interaction time
- temporal gestalt
  - words, gestures
    - » where do they start, the whole matters

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