Why examples are hard, and what to do about it

Examples are important to generate ideas, to test ideas and to communicate ideas, but often we end up talking in generalisations, or read texts that never seem to move from the abstract to the concrete. For the reader and receiver of communication, examples are easy – so why are they so hard to produce?

In this article I'll look at why examples are important, why they are not used, why they are hard to produce … and how to make them easier!

Examples and abstractions

I'm a mathematician, so I love very abstract ideas. The beauty and power of abstraction is both fascinating in itself and intensely practical. I only know of two ways to generalise: through abstraction and through analogy, and even to generalise through analogy requires some level of abstraction in order to understand which features are critical to the analogy.

However, equally important are concrete examples, both real examples from our experience and 'made up' examples from our imagination. These examples themselves may be 'concrete' in a fairly abstract space, such as $2 \times 3 = 3 \times 2$ as an example of commutativity, or may be very solid, such as 'the day I went to Bognor Regis'.

Concrete examples work together with abstractions and theories:

examples **motivate** theories – The real or imagined scenario may exemplify some problem we wish to solve and hence creates the reason why we want to understand a domain better.

examples **inspire** theories – Seeing something in a particular situation may spark those 'why' questions that lead to more abstract investigation.

examples **fuel** theories – Seeing that something is true in several situations suggests that it may be true in general; that is examples are the basis for *induction*.

examples **test** theories – Having, through induction or through reasoning, come up with an abstract idea, we can see whether this holds in different situations. In mathematical terms, abstract arguments are good at universals – showing that something is always true, whereas examples are good at existentials – showing that there is at least one situation in which it is true!

examples **communicate** theories – When we read an abstract description it may be hard to make sense of what the writer means. The example effectively allows us to see each concept in a context.

examples **ground** theories – Perhaps worse than not understanding, we may *think* we understand each other, but in fact the meanings we each connect to concepts may be completely different. Examples serve to ground more abstract discussion, ensuring that our different interpretations at least agree somewhere!

Academics and examples

For years I've wondered why, given the obvious importance of concrete examples, academics are so bad at using them. Textbooks can be pretty bad, but articles and academic monographs are even worse. Strangely, this seems to be more common in the social sciences and humanities than in the sciences or even mathematics.

There are several reasons for this:

(a) **too much understanding** – Writers may simply understand their material so well that they don't realise that the concepts and terms that have become familiar to them are difficult for others, or are simply not the terms others would use.

(b) **too little understanding** – The writer may have a vague idea, but not really understand it clearly enough to be able to make it concrete. This is often an important precursor to deeper understanding, although in this case the attempt to formulate examples can be one of the ways to solidify and deepen understanding.

(c) **fear of misunderstanding** – Examples may over-simplify. When giving an example we often choose a central case, for example; a poster-paint red, not deep crimson, as an example of red. This may mislead a listener into believing it is only the central category, or only the simple case, that we are trying to communicate.

(d) **defence from understanding** – If you stick with vague abstractions, it is hard for anyone to challenge your ideas, but as soon as you give an example, it is easy for someone to say you are wrong.

(e) **rhetoric of incomprehensibility** – By using abstract, hard to understand language a fairly ordinary statement may sound impressive. And if readers do not understand something they often think the ideas must be clever and difficult, not just that the writer is poor at communicating. That is, readers can confuse (a) for (b)!

(f) **being academic** – Researchers may feel that because the product of their work is often abstract theory or concepts, the way they reach this and reason about it must also be abstract. In these circumstances, using examples may appear to be an un-academic way of thinking.

(g) **sounding academic** – For similar reasons, even if the writer has thought about a problem domain concretely themselves, they may choose to write about it purely abstractly, for fear of sounding un-academic, or lightweight. Sadly, this appears to be the 'right' thing to do in terms of academic success: when experimenters submitted the same paper to academic journals in either plain or obscure English, the obscure versions obtained better reviews.

(h) **writing to genre** – In several disciplines the standard way of writing has fossilised into an abstract form of writing. This is the case in mathematical

> proofs, where all the small examples and counter-examples that formed part of the mathematician's creative process are apparently forgotten in the abstract proof. This seems to be part of a cult of minimalism. For different reasons, in parts of the social sciences obscurantism in writing seems to have become the accepted style. Here it appears more that the nuanced complexity of early thinkers in the field has been emulated in form, but not substance, by later writers.

Note that some of these are weaknesses in communication by strong academics (a & c), some failures of weak academics (b) or misguided academics (f & g), and some deceptions of Machiavellian academics (d & e). In practice it is often the first of these that lead to disciplinary patterns of obscurantism (h), but for the most part we see a mixture of many of these reasons.

Interestingly, Newton apparently wrote his *Principia* in geometric terms, rather than the emerging calculus, partly to make it difficult to understand except by those who had sufficient knowledge of the subject. This is not for any of the above reasons, but more a Gnostic-style *writing for the elite* (and by using the work 'Gnostic' I'm aware I am doing the same and sending some readers scurrying for a dictionary!). Arguably the obscurantism of some disciplines is related to this, and certainly part of the rhetoric of abstraction is saying "I can write like this, I am part of the intellectual elite".

Examples are hard

In all the reasons (a – h), the writer is at best poor at communicating, and at worst deliberately misleading. While there are no excuses for the latter, there is in fact a good reason for the former: creating examples is hard.

When an abstract concept or theory arises through induction then the examples come first and so these examples are easy. However, in other cases you really do need to generate or find examples.

Even when the concepts come through induction you may wish to find other examples of the general concept as well as those that drove you to formulate them. For example, in Coleridge's *The Rime of the Ancient Mariner* are the lines 'the furrow followed free'; having read these, you realise that the common first letter gives a sort of inverse rhyme and decide to call this 'alliteration' … it seems obvious to you that alliteration will be a good poetic technique and you try to think back over other poems to recall further examples in addition to the line from the *Ancient Mariner* that started your quest.

If the concepts came through a process of abstract reasoning, then even if you had examples of some of the concepts and theories that started your thinking, you may not have any for the end point of that reasoning. This is often particularly difficult for negative reasoning – 'a place name that begins with A but does *not* end with A'; and pretty hard for conjunctive reasoning – 'a poem that uses alliteration *and* rhyming within a line'.

Furthermore, the concepts may simply have 'come into your mind'. This may be through some more subconscious process of induction or 'reasoning', but if so you are not explicitly aware of the underlying instances that drove the process. For example, poems usually use alliteration within lines and end, or near end, sounds in rhyming. What about a poem where the beginnings of lines sound the same? Let's call this an emhyr (pronounced em-here) … can you think of an example?

The problem in all these cases is that we have a concept and want to either:

(a) generate an example *ex nihilo*, or

(b) recall an example from memory that matches the concept.

It is clear that (a) is difficult, to somehow generate an example of something from the abstract description. In the case of the poetry you would have to create a poem. However, (b) sounds easier. Indeed, this is precisely the annoyance in books or articles that remain abstractly 'in the clouds'. Why not simply give us an example you've seen?, I always think. In fact, even this is not as easy as it sounds.

To understand this we need to think about the way our brains code memories through associations. When I think of 'group theory' all sorts of associations spring to mind, the texture and colours of the first books in which I read about it; Galois, who formalised the area and solved problems outstanding since the Greeks, but died, at nineteen, in a duel; the axioms of the theory (and a few examples!), of course; and Open University presenters with fish-tail ties and flared trousers.

Now, for you, 'group theory' may not mean much. If so, and I explained it to you (which sadly would take longer than explaining alliteration), you would find it hard to think of examples of it, not just because it is mathematical, but because when you have seen real examples (e.g. the manipulations of a Rubik's cube), you will not have thought (consciously or subconsciously) "ah that's connected with group theory". With no associations between your new concept and the old memories, you cannot recall them.

So it is no wonder that we find it hard to recall old examples for new concepts, and perhaps remarkable that we ever manage this at all! In fact the process for recalling old memories for new concepts appears to go something like:

1. You start off *generating* semi-concrete examples of the concept
2. These examples are then available to be matched by *similarity* to past memories (our brains are good at this!)
3. After a while, suddenly an old memory comes to mind (which is a good candidate example, as it is similar to the generated example)
4. You check to see whether it actually matches the concept
5. If it does … hey presto – got it!
6. If it doesn't ... repeat the process starting with new generated examples (step 1) or more retrieved examples (step 3)

Notice how this, in effect, retrieves using analogy, the more

primitive (as in 'older' and more basic) way to generalise.

But also note how this retrieval of past examples, which seemed like the easier process, actually requires that you first generate examples … the difficult process!

Finding examples: transformation and semi-abstractions

As in so many areas, once you understand that examples are difficult, and furthermore why they are difficult, you can start to conceive strategies to make them easier.

Step 6 above says – if the retrieved example doesn't match the concept then repeat the process. In fact, this elides an important step that we may make instead:

6a. if the old example doesn't match the new concept, try to alter it

Finding an example of a concept is not a simple accept/reject decision, but if we find something that is almost right we adapt it.

We may often follow steps 1–6 and 6a unconsciously. However, when we find it difficult to think of examples, we can adopt the process more explicitly. Given that step 1 is the hardest, why not skip it – think of *any* concrete example, analyse why it isn't an example of the concept you are after, and then alter it until it is.

Note that even this process of altering examples normally has starting points that are in some sense roughly in the right area. In the boxed 'emhyr' example I started with a poem, not a mathematical equation. It would be harder to transform $E=mc^2$ into an emhyr! However, when things get really tough this can be a good way of generating novel/different examples; indeed, one of the creativity techniques I suggest is the use of completely random analogies.

So normally there is at least some level of generation of an example, followed by transformation; we have not managed to skip step 1 entirely! However, instead of having to get an example of an abstraction exactly, we are now simply trying to generate examples that are vaguely in the right area: easier for recall (e.g. any poem) and easier for generation.

More 'pure' generation of an example may come through a process of semi-abstracted examples. That is, examples that have concrete elements, but where other parts are still vague or completely unspecified. These can then successively be made more concrete, or may simply suggest or cue a full concrete example.

In the case of the emhyr, we might start with a couplet:

> Looking o'er the troubled sea
> Looming into the greying clouds

or a set of line start sounds and metre only:

> Looking *dumpty dumpty doo*
> Listening *dau dee do da dim*
> Listing gently *dau dau day*
> Looming *didle doble dan*

In the first case we would need to add more lines to make a fuller example, but the couplet alone might be enough to remind us of something. Similarly we might try to complete the line ends in the second example, or simply find that this process of thinking of start sounds reminds us of a poem we have heard before.

In a more analytic domain such as mathematics, we can have similar partially concrete examples: if we were interested in a property of two numbers we might consider what it would be like if the first number was 2. However, these semi-abstractions may also be more descriptive, but using concepts that are well understood. For example, if we have a new property about numbers in general, we may consider what this would be like for even numbers, or positive numbers.

So to do this for real, let's take the idea of an *emhyr*, a poem that alliterates it's initial sounds of lines. Start with any poem, say the most widely known in the English language*:

> I wandered lonely as a cloud
> That floats on high o'er vales and hills,
> When all at once I saw a crowd,
> A host of golden daffodils

Let's try to alter this so that the first words match. 'I' looks like a difficult word to alliterate on (although perhaps 'I wandered' would alliterate with 'Iowa'!), so we can change the word order:

> Lonely I wandered like a cloud
> Floating high o'er hills and vales,
> Lo, when all at once I saw
> Flowing golden daffodils

I've preserved the ABAB pattern from the original and deliberately made it not rhyme (although unintentionally added additional assonance between the 'Lo…' sounds and the 'Flow…' sounds). This may not be good poetry, but it may serve at least as an example to talk about, and furthermore act as a cue and remind you of a real poem that does this.

Actually I think one of the discipline genre issues in writing about poetry is that made-up examples like this are inevitably bad poetry (and worse when bowdlerising good poetry!), not least because an effective example will have the features you are trying to show and nothing else, whereas one of the aspects of good poetry is precisely that multiple poetic mechanisms (word sound, imagery, rhythm) work together. In mathematics trivial examples are more acceptable.

**Daffodils*, William Wordsworth, 1804

or even … http://www.golakes.co.uk/wordsworthrap/

Working in the space between

We initially started with a dichotomy between abstraction and concrete examples. However, the process of semi-abstraction reminds us that in fact all our mental images are somewhat like this. When you recall a face, it is not every line and feature you recall, but parts and general aspects; indeed our very perceptual systems have already done a level of abstraction. Perhaps it is only when we externalise these, whether in action or in communication, that we start to make them truly concrete, but even then our words themselves are highly abstracted (e.g. the word 'poem' or even 'cloud' covers so many things).

In mathematics, the most concrete things are themselves abstractions (e.g. numbers), and through the process of naming and axiomatising, more and more complex theoretical constructs become in some sense 'concrete', but this simply mirrors the 'normal' process of day-to-day language. The flow between more and less concrete examples is fairly fluid and often we do not need fully elaborated concrete examples to get inspiration for where to go next.

Similar levels of inspiration and reasoning can happen in this in-between space of semi-abstracted examples in other domains. For example, as I thought about the semi-abstracted emhyrs, it became obvious that the technique (if it works at all) would be most effective in simple patterns, perhaps three lines starting with 'lo…' words, or three lines starting with 'fl…' words.

Challenge! Write an emhyr about HCI and mail it to alan@hcibook.com. I'll post the best on www.alandix.com/blog and at the HCI conference in Lancaster.